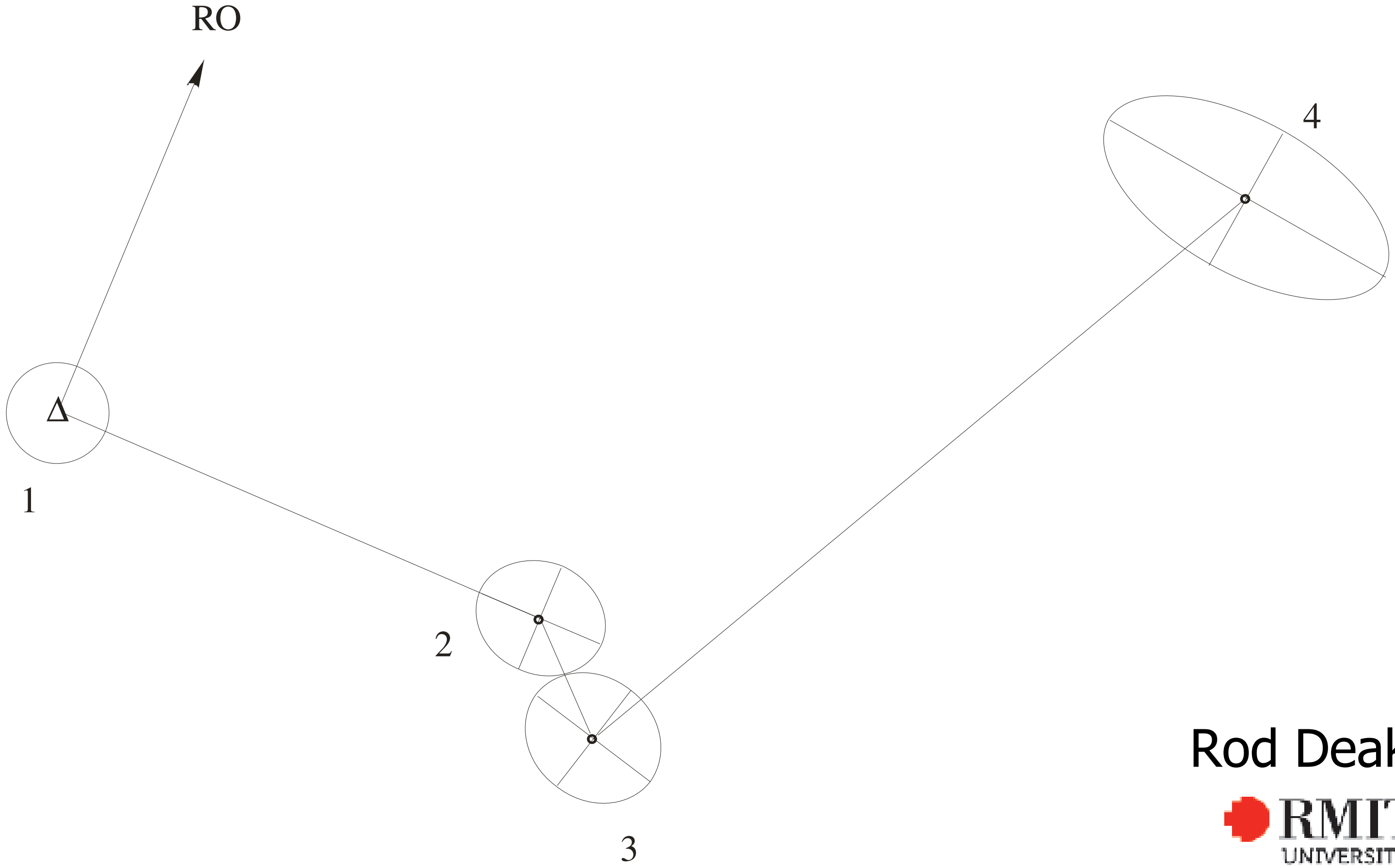
TRAVERSE ANALYSIS



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Traverses and Traversing

- Traverse
  - A fundamental survey operation.
  - A framework of lines – measured bearings and distances.
- Property corners; fences; buildings; etc. are connected to traverses.
- Areas of land; cadastral boundaries; defined from traverse measurements.

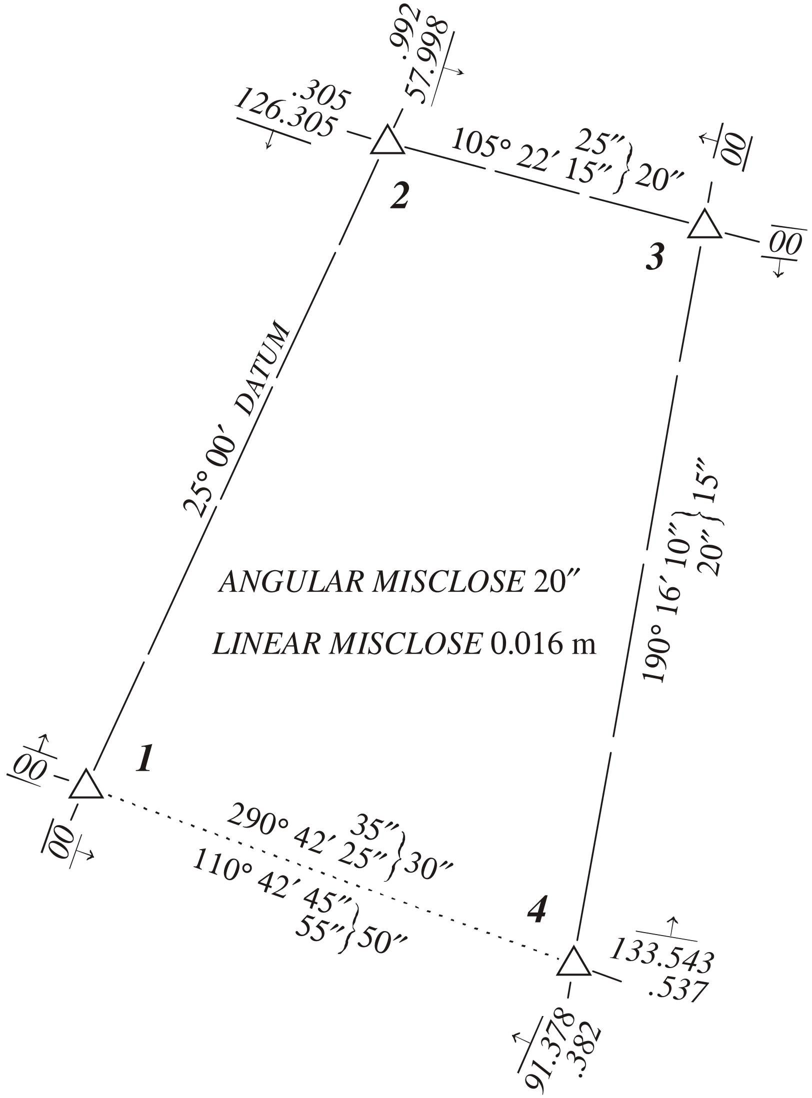
Traverses and Traversing

- Closed traverses:
  - Start and finish at same point;
  - Have linear and angular misclosures that are indicators of traverse quality.
- Open traverses:
  - Start and finish at different points;
  - Have misclosures only if the terminal points are ‘known’.
- This presentation is concerned with closed traverses.

Traverses and Traversing

- <u>Traverse network</u>
  - Several closed traverses.
  - Analysed using sophisticated least squares theory and computer software.
- This presentation is not about traverse networks or least squares.

Single Closed Traverse



Is this traverse OK ?

Are angular and linear misclosures OK ?

Only <u>three</u> redundant measurements

- Bearing: 1 $\rightarrow$ 4
- Bearing: 4 $\rightarrow$ 1
- Distance: 1 $\rightarrow$ 4

Rule for assessing Quality

- Use Propagation of Variances to:
  - Estimate standard deviation of bearing of closing line;
  - Estimate standard deviation of distance of closing line.
- **Reject** traverse if misclosures are greater than **twice** estimated standard deviations.

Propagation of Variances

- A mathematical rule for estimating effects of random measurement errors
- Say $E = L\sin\theta$ and measurements L and θ have error variances $s^2{}_L$ and $s^2{}_\theta$ and covariance $s_{L\theta}$ then

$$s^2{}_E = (\sin\theta)s^2{}_L + (L\cos\theta)s^2{}_\theta + 2(L\sin\theta\cos\theta)s_{L\theta}$$

- Covariances = zero for independent measurements.

Propagation of Variances

- This rule conveniently expressed using matrix algebra.
- If $\mathbf{y} = f(\mathbf{x})$ then $\mathbf{Q}_{yy} = \mathbf{J}_{yx}\, \mathbf{Q}_{xx}\, \mathbf{J}^{T}_{yx}$
  - $\mathbf{y}$ is a vector of computed quantities
  - $\mathbf{x}$ is a vector of measurements
  - $\mathbf{Q}_{yy}$ $\mathbf{Q}_{xx}$ are variance matrices
  - $\mathbf{J}_{yx}$ is a matrix of partial derivatives
- Very easy for computers.

Precision of traverse distances

- $s_d = \pm(A + B \times d / 1000)$
  - s_d is estimated standard deviation
  - d is traverse distance in metres
  - A is in mm
  - B in ppm (ppm = mm per km)
- Example:
  - $s_d = \pm(5 \text{ mm} + 3 \text{ ppm})$ if $d = 1000\,\text{m}$ then $s_d = \pm 8.0$ mm

Precision of traverse bearings

- $s^2_{For} = s^2_{Back} + s^2_{\beta}$
  - s^2_{For} is variance of forward bearing
  - s^2_{Back} is variance of back bearing
  - s^2_{β} is variance of traverse angle
- $s^2_{\beta} = s^2_{PR} + s^2_{CENT}$
  - s^2_{PR} is variance of pointing and reading to target (known)
  - s^2_{CENT} is variance of instrument and target centring errors (formula)

Precision of centring errors

Several candidates

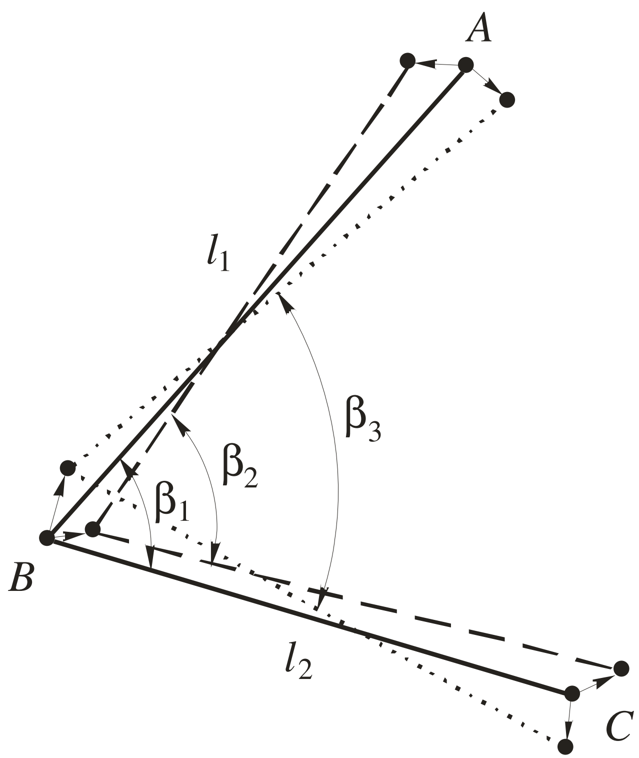
average error due to imperfect centring	$= r_c \dfrac{2}{\pi}\sqrt{\dfrac{1}{l_1^2}+\dfrac{1}{l_2^2}-\dfrac{2\cos\beta}{l_1 l_2}}$	Briggs (1912)
probable error due to imperfect centring	$= p_c \sqrt{2\pi}\sqrt{\dfrac{1}{l_1^2}+\dfrac{1}{l_2^2}+\dfrac{\cos\beta}{2 l_1 l_2}}$	Miller (1936)
standard deviation due to imperfect centring	$= s_c \sqrt{2}\sqrt{\dfrac{1}{l_1^2}+\dfrac{1}{l_2^2}-\dfrac{\cos\beta}{l_1 l_2}}$	Richardus (1966)

Briggs: calculus (instrument only)

Miller: calculus

Richardus: simplified propagation of variances

Monte Carlo angle simulation



- Calculate β_1 from *A, B, C*
- Move *A, B, C* small random amounts
- Calculate β_2 from *A, B, C*
- Repeat process for the angles $\beta_1, \beta_2, \beta_3, \ldots, \beta_n$
- Determine standard deviation of set of angles

Precision of centring errors

A better formula from Monte Carlo simulation

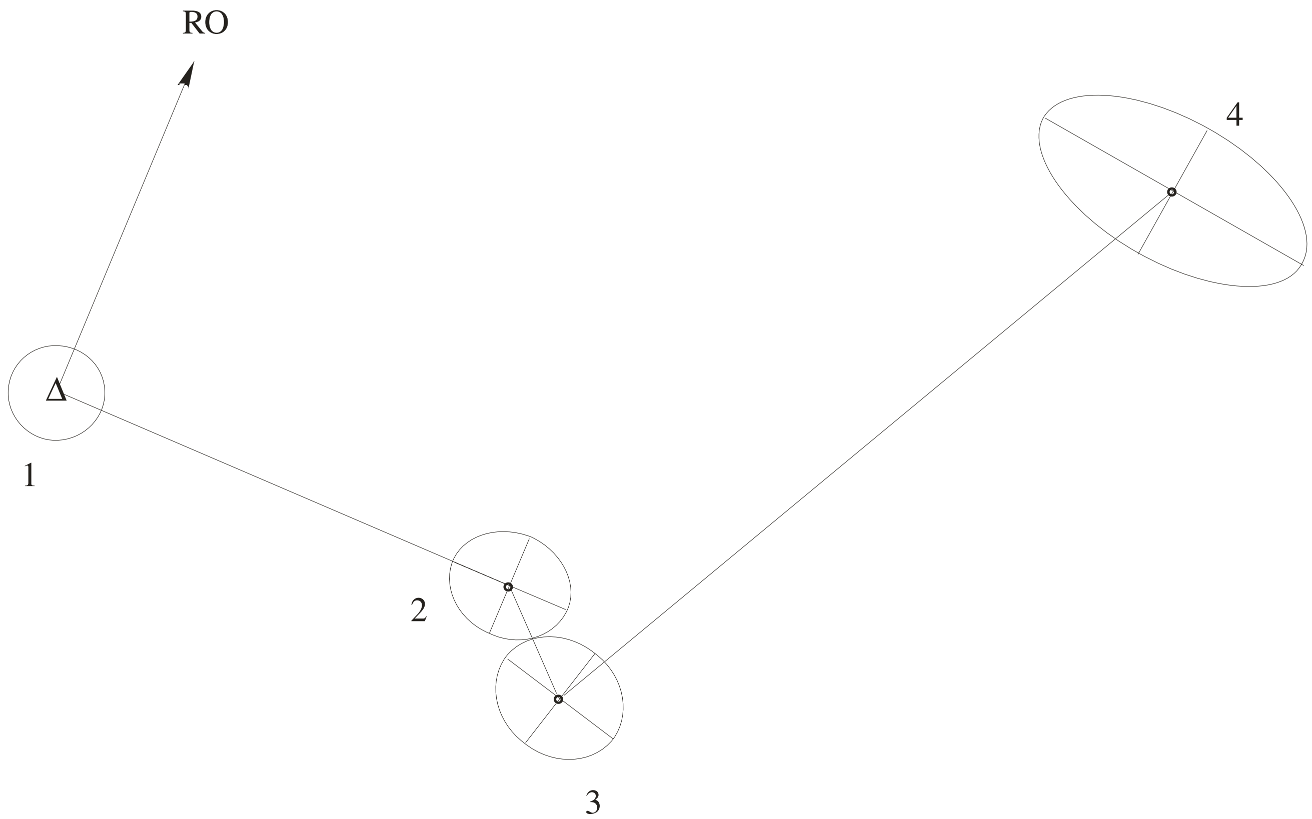
$$\text{standard deviation due to imperfect centring } = s_c\sqrt{\frac{1}{l_1^2}+\frac{1}{l_2^2}-\frac{\cos\beta}{l_1 l_2}}$$ Deakin (2012)

Estimating traverse quality

Using the rules for precision of traverse bearings and distances, and propagation of variances, allows the estimation of precisions of the closing line of a traverse.

These can be compared with actual misclosures to assess the quality of a traverse.

Traverse analysis



These new rules are a modern approach to quality control. The END